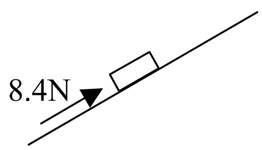
MECHANICS (C) UNIT 1   TEST PAPER 3

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. Two forces **P** and **Q**, of magnitudes 7 N and 24 N respectively, act in directions which are perpendicular to each other. Calculate the magnitude of the resultant force and the angle that this resultant makes with the line of action of **P**. [4]

2. Two particles A and B, of masses 0·3 kg and 0·7 kg, are moving towards each other on a smooth surface along the same straight line, with speeds 9 ms^{-1} and $2u$ ms^{-1} respectively. They collide directly. Immediately after the collision A and B both move together in the same direction with speed u ms^{-1}. Find the two possible values of u. [6]

3. A particle P, initially at rest at the point O, moves in a straight line such that at time t seconds after leaving O its acceleration is $(12t - 15)$ ms^{-2}. Find
   (i) the velocity of P at time t seconds after it leaves O, [3]
   (ii) the value of t when the speed of P is 36 ms^{-1}. [3]

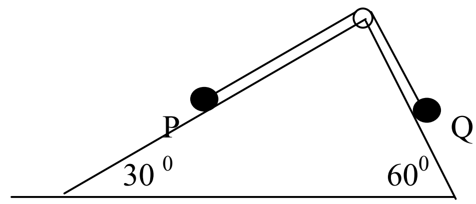
4. 

A small packet, of mass 1·2 kg, is at rest on a rough plane 8.4 inclined at an angle α to the horizontal. The coefficient of friction between the packet and the plane is 1/8 .

When a force of magnitude 8·4 N, acting parallel to the plane, is applied to the packet as shown, the packet is just on the point of moving up the plane. Modelling the packet as a particle,
   (i) show that $7(\cos\alpha + 8\sin\alpha) = 40$. [5]
   Given that the solution of this equation is $\alpha = 38^0$,
   (ii) find the acceleration with which the packet moves down the plane when it is released from rest with no external force applied. [4]

5. A car moves in a straight line from P to Q, a distance of 420 m, with constant acceleration. At P the speed of the car is 8 ms^{-1}. At Q the speed of the car is 20 ms^{-1}. Find
   (i) the time taken to travel from P to Q, [2]
   (ii) the acceleration of the car, [2]
   (iii) the time taken for the car to travel 240 m from P. [3]
   Given that the mass of the car is 1200 kg and the tractive force of the car is 900 N,
   (iv) find the magnitude of the resistance to the car's motion. [3]

6. Two particles P and Q, of masses 3 kg and 2 kg respectively, rest on the smooth faces of a wedge whose cross-section is a triangle with angles 30^0, 60^0 and 90^0, as shown. P and Q are connected by a light inextensible string, parallel to the lines of greatest slope of the two planes, which passes over a smooth fixed pulley at the highest point of the wedge.



The system is released from rest with P 0·8 m from the pulley and Q 1 m from the bottom of the wedge, and Q starts to move down. Calculate

(i) the acceleration of either particle, [4]

(ii) the tension in the string, [2]

(iii) the speed with which P reaches the pulley.

[2]

Two modelling assumptions have been made about the string and the pulley.

(iv) State these two assumptions and briefly describe how you have used each one in your solution. [4]

7. Two stones are projected simultaneously from a point O on horizontal ground. Stone A is thrown vertically upwards with speed 98 ms^{-1}. Stone B is projected along the smooth ground in a straight line at 24·5 ms^{-1}.

(i) Find the distances of the two stones from O after t seconds, where $0 < t < 20$. [3]

(ii) Show that the distance d m between the two stones after t seconds is given by

$$d^2 = 24{\cdot}01\,(t^4 - 40t^3 + 425t^2).$$ [5]

(iii) Hence find the range of values of t for which the distance between the stones is decreasing.

[5]

MECHANICS 1 (C) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1. Resultant $=\sqrt{(7^2 + 24^2)} = 25$ N   Angle $= \tan^{-1}(24/7) = 73{\cdot}7^0$ M1 A1 M1 A14

2. $0{\cdot}3 \times 9 + 0{\cdot}7 \times -2u = (0{\cdot}3 + 0{\cdot}7) \times u$ $2{\cdot}4u = 2{\cdot}7$ $u = 1{\cdot}125$ M1 A1 A1
or $0{\cdot}3 \times 9 + 0{\cdot}7 \times -2u = (0{\cdot}3 + 0{\cdot}7) \times -u$ $0{\cdot}4u = 2{\cdot}7$ $u = 6{\cdot}75$ M1 A1 A1   6

3. (i) $v = $ š $a\,dt = 6t2 - 15t + c$ $v(0) = 0 : c = 0$ $v = 6t2 - 15t$ M1 A1 A1
(ii) $6t^2 - 15t - 36 = 0$ $3(2t + 3)(t - 4) = 0$ $t = 4$ M1 A1 A1   6

4. (i) Resolve perp. and // plane: $R = 1{\cdot}2g\cos\alpha$, $8{\cdot}4 = 1{\cdot}2g\sin\alpha + \frac{1}{8}R$ M1 A1 A1
$1{\cdot}2g(\sin\alpha + \frac{1}{8}\cos\alpha) = 8{\cdot}4$ $7(8\sin\alpha + \cos\alpha) = 40$ M1 A1
(ii) Acc. down plane $= g\sin 38$Ý $- \frac{1}{8}g\cos 38 = 5{\cdot}07$ ms^{-2} M1 M1 A1 A1   9

5.	(i) $420 = \frac{1}{2}(20 + 8)t$ $t = 30$ s	M1 A1	
	(ii) $20 = 8 + 30a$ $30a = 12$ $a = 0{\cdot}4$ ms^{-2}	M1 A1	
	(iii) $s = ut + \frac{1}{2}at^2$: $240 = 8t + 0{\cdot}2t^2$ $t^2 + 40t - 1200 = 0$	M1	
	$(t - 20)(t + 60) = 0$ $t = 20$	M1 A1	
	(iv) $F = ma$: $900 - R = 1200(0{\cdot}4)$ $R = 900 - 480 = 420$ N	M1 A1 A1	10
6.	(i) $2g \cos 30 - T = 2a$, $T - 3g \cos 60 = 3a$	B1 B1	
	Add : $g(\sqrt{3} - 1{\cdot}5) = 5a$ $a = 0{\cdot}455$ ms^{-2}	M1 A1	
	(ii) $T = 3a + 1{\cdot}5g = 16{\cdot}1$ N	M1 A1	
	(iii) $v^2 = u^2 + 2as = 0 + 2a(0{\cdot}8) = 0{\cdot}728$ $v = 0{\cdot}853$ ms^{-1}	M1 A1	
	(iv) String inextensible, so acceleration the same for both particles	B1 B1	
	Pulley smooth, so tension is constant throughout the string	B1 B1	12
7.	(i) $s_A = 98t - 4{\cdot}9t^2$ $s_B = 24{\cdot}5t$	M1 A1 B1	
	(ii) $d^2 = (4{\cdot}9t(20 - t))^2 + (24{\cdot}5t)^2 = 4{\cdot}92(t^2(t^2 - 40t + 400) + (5t)^2)$	M1 A1 A1	
	$= 24{\cdot}01t^2(t^2 - 40t + 400 + 25) = 24{\cdot}01(t^4 - 40t^3 + 425t^2)$	M1 A1	
	(iii) $\frac{\mathrm{d}}{\mathrm{dt}}(\mathrm{d}^2) = 24{\cdot}01(4t3 - 120t^2 + 850t) < 0$ for decreasing function	M1 A1	
	When $4t^2 - 120t + 850 = 0$, $t = 11{\cdot}5$ or $t = 18{\cdot}5$, so range is	M1 A1 (both)	
	$11{\cdot}5 \le t \le 18{\cdot}5$	A1	13